Studying Strong and EW Interactions using Electron Scattering at JLab

Xiaochao Zheng
September 21, 2007

Introduction — nucleon structure and electron scattering

Three types of DIS
  - Unpolarized DIS and what is the nucleon made of;
  - Polarized DIS and what forms the nucleon spin;
  - Parity Violating DIS and what we will learn from it?

PVDIS program at Jefferson Lab
  - Program at the 12 GeV Upgrade
  - E05-007 using a 6 GeV beam — your opportunity

Summary

X. Zheng, September 2007, Research talk at Univ. of Virginia
## Four Interactions of Our Nature

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Force Constant</th>
<th>Theory</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$10^{-38}$</td>
<td>General relativity</td>
<td>Well understood at large distances</td>
</tr>
<tr>
<td>Electro-Magnetic</td>
<td>1/137</td>
<td>EM: Fully understood</td>
<td></td>
</tr>
<tr>
<td>Weak</td>
<td>$10^{-5}$</td>
<td>Weak: Well understood at current collider energies, <em>but has room for New Physics beyond the EW Standard Model</em></td>
<td></td>
</tr>
<tr>
<td>Strong</td>
<td>&lt;1</td>
<td>SU(3), QCD</td>
<td>Poorly understood, cannot calculate</td>
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**Strong Interaction:**

- Asymptotic freedom/Confinement;
- Mostly non-perturbative – difficult to handle theoretically;
- Where and how can we prove that QCD is the correct theory?
  - Hadron structure as an ideal laboratory
  - DIS: testing ground of QCD in the perturbative regime.

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X. Zheng, September 2007, Research talk at Univ. of Virginia
Electrons ($\mu$'s) interact with the target by exchanging a “virtual” photon;

Two variables to describe how the target behave: $1/Q^2$ and $\nu$;
Exploring Nucleon Structure Using EM Probe

The cross section:

\[ \frac{d^2 \sigma}{d \Omega dE'} = \sigma_{\text{Mott}} \left[ \alpha F_1(Q^2, \nu) + \beta F_2(Q^2, \nu) \right] \]

For point-like target

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Exploring Nucleon Structure Using EM Probe (cont.)

- Elastic – the nucleus appears as a rigid body: \( Q^2 = 2M_T \nu \)
- Quasi-elastic – individual nucleon appears as a rigid body; nucleus = incoherent sum of nucleons: \( Q^2 = 2M_N \nu \)

Nucleon form factors well measured, but need QCD-based calculations to understand—very difficult (non-perturbative)

X. Zheng, September 2007, Research talk at Univ. of Virginia
Resonance region – quarks inside the nucleon react coherently

For resonances, typically use phenomenological models to study N-N* transition amplitudes and polarizations.

— Have a project to extract polarization variables of N-N* transition using existing JLab Hall B data (will mention briefly at the end)
Exploring Nucleon Structure Using EM Probe (cont.)

- Deep Inelastic Scattering (DIS):
  - Quarks start to react incoherently
  - Start to see constituents of the nucleon

For DIS, perturbation theory starts to work, can test perturbative QCD.

Since the early 1960's, there have been two types of DIS

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Before DIS (pre -1960)

- 1933: Protons are not point-like \((\kappa_p = 1.79)\);
- 1950's: Nucleons have a structure;

O. Stern — "for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"

R. Hofstadter — "for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the structure of the nucleons" (shared with R. L. Mössbauer)

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"First Generation": Unpolarized DIS (1960~1980)

- 1968: First DIS data from SLAC
  - Nucleons have hard point-like scattering centers (partons);
  Friedman, Kendall, Taylor et al. 1990

- 1969: Prediction for the scaling behavior: Bjorken
  As $Q^2, \nu \to \infty$, and $Q^2/\nu$ fixed (Bjorken scaling variable $x_{Bj} = Q^2/2M\nu$)

- Observed scaling experimentally

- Quark-Parton model (QPM): Feynman
  $F_{1,2}(x, Q^2) \to F_{1,2}(x)$
  - DIS = incoherent sum of e scattering off asymptotically free quarks
  - $x_{bj} = \text{fraction of the nucleon momentum carried by the struck quark};$

"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

X. Zheng, September 2007, Research talk at Univ. of Virginia
"First Generation": Unpolarized DIS (1960~1980)

"for the discovery of asymptotic freedom in the theory of the strong interaction"

\[
\alpha_s(Q^2) = \frac{4\pi}{(11 - 2n_f/3) \ln(Q^2/\Lambda^2)}
\]

1972-1973: 

- Asymptotic freedom
  - Gross, Wilczek & Politzer 2004

- QCD became a possible (and the leading) theory for the strong interaction.

- Unpolarized DIS has been the major experimental tool to study unpolarized nucleon structure and a testing ground for perturbative QCD.

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\[
\frac{d^2 \sigma}{d \Omega \, dE'} = \sigma_{\text{Mott}} \left[ \alpha F_1(Q^2, \nu) + \beta F_2(Q^2, \nu) \right]
\]
After 39 years of DIS experiments, the unpolarized structure of the nucleon is reasonably well understood (for moderate $x_{Bj}$ region).
“Second Generation”: Polarized DIS (1980~present)

- Scattering cross section is spin-dependent (imaging throwing two small magnets together)

- Longitudinal

\[
\frac{d^2 \sigma^{\uparrow\downarrow}}{d \Omega \, dE'} - \frac{d^2 \sigma^{\uparrow\uparrow}}{d \Omega \, dE'} \propto \sigma_{\text{point-like}} \left[ \alpha' g_1(x, Q^2) + \beta' g_2(x, Q^2) \right]
\]

- Transverse

\[
\frac{d^2 \sigma^{\uparrow\leftarrow}}{d \Omega \, dE'} - \frac{d^2 \sigma^{\uparrow\rightarrow}}{d \Omega \, dE'} \propto \sigma_{\text{point-like}} \left[ \alpha'' g_1(x, Q^2) + \beta'' g_2(x, Q^2) \right]
\]
Polarized Structure Function and the Nucleon Spin Structure

- in QPM and the infinite momentum frame:

\[ g_1(x) = \frac{1}{2} \sum e_i^2 [q_i^+(x) - q_i^-(x)] = \frac{1}{2} \sum e_i^2 [\Delta q_i(x)] \]

- The integral of \( g_1(x) \) over \( x \) describes how much of the nucleon's spin is carried by quarks' spin
“Second Generation”: Polarized DIS (1980~present)

*Spin observables provide new testing ground of QCD and our understanding of the nucleon structure*

- 1988-1989: data showed quarks' spin contributes (12±17)% to the proton spin — “Spin Crisis”

- Suspected: the sea quarks (mainly strange quarks) may carry a large fraction ... (program on PV elastic electron scattering)

- Current understanding of the nucleon spin:
  \[
  \frac{1}{2} = S^N_Z = S^q_Z + L^q_Z + J^q_Z
  \]

  - Quark spin contributes about 30% to the nucleon spin
  - Strange (and other sea) quarks do not contribute significantly

- Polarized DIS has served as a major experimental tool to study the nucleon spin structure and further testing of pQCD. *(This is also one of my research interest since my PhD work, however I will not talk about it today)*
Current Knowledge of Nucleon Polarized Structure

(after 20 years of study)

\[ g_1(x) = \frac{1}{2} \sum e_i^2 [q_i^+(x) - q_i^-(x)] = \frac{1}{2} \sum e_i^2 \Delta q_i(x) \]

\[ g_{1p}(x) = \frac{1}{2} \sum e_i^2 [q_i^+(x) - q_i^-(x)] = \frac{1}{2} \sum e_i^2 \Delta q_i(x) \]
“Third Generation”: Parity Violating DIS (today~)

- EM observables — $\sigma, A...$ (polarized beam + polarized target)
  - hadron structure, strong interaction and its standard model (QCD);

- Weak observables — parity violating asymmetries ($A_{PV}$)
  (polarized beam + unpolarized target)

\[ A_{PV} = \]

\[ A_{LR} = \frac{\sigma^r - \sigma^l}{\sigma^r + \sigma^l} \approx \frac{Q^2}{M_Z^2} \approx 120 \text{ ppm} \]

at $Q^2 = 1 (\text{GeV}/c)^2$

- study hadron structure
  - elastic scattering: strange form factors
  - DIS: non-perturbative effects, CSV etc...

- test the standard model of electro-weak interaction

**A4, G0, HAPPEX, SAMPLE**

**PVDIS**

**Qweak**

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ElectroWeak Standard Model

- SM works well at present energy range;
- Conceptual reasons for new physics:
  What happens in the “high-energy desert”?

Data exist: cannot be explained by the SM ($m_\nu$, NuTeV anomaly...);
ElectroWeak Standard Model

- SM works well at present energy range;
- Conceptual reasons for new physics:
  What happens in the “high-energy desert”? 

indirect:
NuTeV, Qweak, PV-DIS

Hi-E direct:
LEP, LHC

(250 GeV ~ 5 \times 10^{14} \text{ GeV} ~ 2.4 \times 10^{18} \text{ GeV})?

- Search for Physics beyond the Standard Model

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Testing the EW Standard Model – Running of $\sin^2 \theta_W$ and the NuTeV Anomaly
Test of EW Standard Model Using PVDIS

\[ A_{PV} = \]

For a deuterium target

\[ A_d = (540 \text{ ppm}) Q^2 \frac{2 C_{1u}[1+R_C(x)] - C_{1d}[1+R_S(x)] + Y(2 C_{2u} - C_{2d}) R_V(x)}{5 + R_S(x) + 4 R_C(x)} \]

\[ C_{1u} = g_{A}^e g_{V}^u = -\frac{1}{2} + \frac{4}{3} \sin^2(\theta_W) \]

\[ C_{2u} = g_{V}^e g_{A}^u = -\frac{1}{2} + 2 \sin^2(\theta_W) \]

\[ C_{1d} = g_{A}^e g_{V}^d = \frac{1}{2} - \frac{2}{3} \sin^2(\theta_W) \]

\[ C_{2d} = g_{V}^e g_{A}^d = \frac{1}{2} - 2 \sin^2(\theta_W) \]

From \( A_d \) can extract \( C_{1,2q} \) and \( \sin^2 \theta_W \).

In the SM, tree level
Test of EW Standard Model Using PVDIS


- Development in experimental technique allows to search for new physics

$$A_{PV} = \left[ \begin{array}{c} e \gamma e \\text{moller (slac E158, completed)}: \text{pure-leptonic} \\
\text{qweak (JLab Hall C, future)}: \text{only } C_{1q}'s \\
\text{Only PVDIS is sensitive to } C_{2q}'s (\text{new physics in the quark axial coupling}) \\
\end{array} \right]$$

- PVDIS is complementary to other SM tests:

  - Moller (SLAC E158, completed): pure-leptonic
  - Qweak (JLab Hall C, future): only $C_{1q}$'s
  - Only PVDIS is sensitive to $C_{2q}$'s (new physics in the quark axial coupling)
Study the Nucleon Structure Using PVDIS

For a deuterium target

\[ A_d = (540 \text{ ppm}) Q^2 \left( \frac{2C_{1u} [1 + R_C(x)] - C_{1d} [1 + R_S(x)] + Y (2C_{2u} - C_{2d}) R_V(x)}{5 + R_S(x) + 4R_C(x)} \right) \]

\( R_C(x), R_S(x), R_V(x) \) are related to parton distribution functions, thus \( A_d \) is sensitive to:

- Non-perturbative QCD effects
- Strange sea asymmetry \( s(x) - \bar{s}(x) \neq 0 \)
- Charge symmetry violation (compare to \( A_p \)) \( u^p(x) \neq d^n(x) \)
  \( d^p(x) \neq u^n(x) \)

All these are not yet calculable in QCD!

Direct observation (and precision measurement) of any one will provide more stringent tests on QCD – Standard Model of the strong interaction

X. Zheng, September 2007, Research talk at Univ. of Virginia
JLab 6 GeV Experiment 05-007

Co-spokesperson & contact:  X. Zheng
Co-spokesperson: P.E. Reimer, R. Michaels

(Hall-A Collaboration Experiment, approved by PAC27, rated A-)

- PVDIS has not been done again since Prescott's experiment;
- We will start this program at JLab, with the 1st experiment E05-007 to run in 2009:
  - Use 85μA, 6 GeV, 80% polarized beam on a 25-cm LD2 target;
  - Two Hall A High Resolution Spectrometers detect scattered electrons;
  - Measure $A_d$ at $Q^2=1.10$ and 1.90 GeV$^2$ to about 2% (stat.);

ANL, Calstate, FIU, Jlab, Kentucky, U. of Ljubljana (Slovenia), MIT, UMD, UMass, UNH, Universidad Nacional Autonoma de Mexico, Rutgers, Smith C., Syracuse, UVa, W&M

X. Zheng, September 2007, Research talk at Univ. of Virginia
E05-007 Physics Goals (I)

\[ A_d = \left( \frac{3 G_F Q^2}{\pi \alpha 2 \sqrt{2}} \right) \frac{2 C_{lu} \left[ 1 + R_C(x) \right] - C_{ld} \left[ 1 + R_S(x) \right] + Y \left( 2 C_{2u} - C_{2d} \right) R_V(x)}{5 + R_S(x) + 4 R_C(x)} \]

- From \( A_d \) at \( Q^2=1.90 \) (GeV/c)\(^2\), can extract \( 2C_{2u} - C_{2d} \) to ±0.03 (if assuming small hadronic effects) — order of magnitude improvement compared to world data (PDG);
- provide constraints on new physics & tests of the Standard Model up to 1 TeV mass limit;
  - \( Z' \) Searches
  - Compositeness (4-fermion contacts)
  - Leptoquarks.

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E05-007 Physics Goals (II)

- Separation of new physics beyond the SM and hadronic effects is difficult – can be investigated by choosing proper kinematics.

- $A_d$ at $Q^2=1.10 \text{ (GeV/c)}^2$ will help to constrain the hadronic non-perturbative effects:
  - Help to secure the $C_2$ SM test at 1.9 $\text{(GeV/c)}^2$;
  - Will be possibly the first observation of non-perturbative effects in PV process;
  - Will help to investigate the non-perturbative contribution to the NuTeV anomaly.
E05-007 and the Ultimate Goal of PVDIS Program

- A full separation of 1) New Physics beyond the SM and 2) hadronic effects is beyond the reach of E05-007, currently planning to have a dedicated large acceptance device, combined with JLab upgraded 12 GeV beam to do so – time scale: ~2015

- Nevertheless, E05-007 will provide the first precision data on PVDIS Ad

  - If a deviation from SM value is observed, definitely indicate something interesting – either direct observation of CSV, strange sea asymmetry (testing QCD – SM of strong interactions); or of new physics beyond the SM of electroweak interactions;

  - Provide important guidance to the 12 GeV/large acceptance program in the next decade.
The Accelerator (CEBAF)
We are planning a day trip to JLab early next year – free food, meet physicists and students – will be quite fun! stay tuned!
Experimental Hall A

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PV-DIS (E05-007) Requirements

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PV-DIS (E05-007) Requirements

- upgrade from IR to green laser to provide 1% precision
- understand polarized source-related systematics
- upgrade to flash-ADC based DAQ to count 1MHz rate
Opportunity for Young Scientists

- Opportunities exist for 1-2 new graduate students – *that means you!*

X. Zheng, September 2007, Research talk at Univ. of Virginia
Opportunity for Young Scientists

- Opportunities exist for 1-2 new graduate students – *that means you!*

- Starting June 2008 (or earlier), participate in:
  - Building the fast counting DAQ (great training on electronics and DAQ);
  - Compton upgrade (laser optics, vacuum, control etc.):
  - Understand polarized source systematics (become an expert on running PV experiments)
  - Observe (and know) how to run an experiment at JLab!
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- Running the experiment (2009)
  - Get your Ph.D. data.

Other personnel: Ramesh Subedi (postdoc), and many other experts
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- Analyzing data and publishing the results (2009-2010)
  - Parity experiments typically have shorter analyzing time;

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- Running the experiment (2009)
  - Get your Ph.D. data.

- Analyzing data and publishing the results (2009-2010)
  - Parity experiments typically have shorter analyzing time;

- Write your thesis and graduate! (2010-2012?)
  - Your thesis will become the foundation of the new JLab PVDIS program.

Other personnel: Ramesh Subedi (postdoc), and many other experts

X. Zheng, September 2007, Research talk at Univ. of Virginia
Summary (3rd last slide)

- A new DIS program – PVDIS – is being launched at Jefferson Lab
  - Test Electro-weak Standard Model
  - Study nucleon structure beyond current QCD
  - First step is to do a 6 GeV measurement (E05-007)
  - Will be one of the leading experiments of JLab 12 GeV

- E05-007 will provide data sufficient for 1-2 (excellent) Ph.D. thesis, within a decent time

- Talk to me (xz5y@) or stop by my office anytime to know more! (even if you just want to chat!)
Project on Polarized N-N* Transition (2nd last slide)

- JLab Hall B (CLAS) collaboration approved proposal (spokesperson) – Ph.D. topic possible
  - Analyze CLAS eg4 data (already on tape)
  - Data analysis may take longer time than PVDIS
  - Need additional hardware project to complete a Ph.D. (likely: work on polarized NH3/ND3 target with Prof. Day and Prof. Crabb)

- Again, talk to me anytime!

Other UVa students doing similar Hall B projects: Josh Pierce, Kangkang Li
How a good Ph.D can help with Your Future (last slide)

You may be still “shopping around” for research now,
- Your immediate goal is to pass your qualify ASAP (if you haven’t yet)
- But it’s good to start thinking early (I chose my advisor during the 1st week of arriving at MIT)

A good Ph.D. work is the foundation of your future:
- If you are interested in academia/research, need good reference letters for future job hunting, and you will start building your “collaboration network” as early as now;
- If you are interested in going to industry/financial – again good letter are the key. And believe it or not, they don’t care whether you did PV-DIS or other physics, good discipline and problem solving is what they are looking for, and you will get trained for these by working (hard) on your Ph.D. So just make your choice, and then love and work hard on it - Of course, I wish you still choose PVDIS/nuclear exp phys as your research.

I myself received strong support since the 1st day in the US, and I will try to return this to you

X. Zheng, September 2007, Research talk at Univ. of Virginia
Became operational in May 2006;
Future of PV-DIS at JLab

- E05-007 started the PV-DIS program @ JLab:
  - SM test, precision $C_{2q}$ and $\sin^2\theta_W$
    (complimentary to other HEP and Nuclear experiments)
  - Study of hadronic effects:
    - Higher-twist study (low $Q^2\,\alpha_S$, confinement);
    - On a proton target: measure PDF ratio $d/u$ at high $x$;
    - Measure Charge Symmetry Violation (CSV).

- Leading program @ 12 GeV upgrade (2015?~), two initiatives:
  - New large acceptance solenoid device – enables a clean separation of New EW Physics, CSV, and non-perturbative effects.
Some basics about the nucleon:

- In high school physics, N = 3 quarks
- A more complete picture: Nucleon =
  - three “valence” quarks
  - a large amount of $q\overline{q}$ pairs (“sea” quarks) from the vacuum
  - gluons that “glue” all quarks together (strong force)
- A little more advanced: 3 valence $q$
  “dressed up” by sea quarks and gluons = 3 constituent quarks
Jlab 12 GeV

- Add new hall
- Upgrade magnets and power supplies
- Add 5 cryomodules
- 20 cryomodules
- CHL-2
- Add arc
- Add 5 cryomodules
- 20 cryomodules
- Enhance equipment in existing halls

X. Zheng, September 2007, Research talk at Univ. of Virginia
About PV Elastic Scattering

Interest in PV electron elastic scattering started during the time of the “proton spin crisis” - Do strange quarks contribute to the nucleon spin (and how about the nucleon mass...)?

Several experiments were performed, found: small $G_E^s$, $G_M^s$ – small strangeness contributions to properties of the nucleon A4, G0, HAPPEX, SAMPLE

Now as the proton spin crisis becomes more understood, PV elastic scattering program is close to its end.
Scaling Violation in QCD

- Bjorken limit: $Q^2 \rightarrow \infty$, $x_{Bj}$ fixed, (strict) one photon exchange
  
  \[ 1/Q \]

  no scale ($Q^2$) dependence, scaling

- High $Q^2$, soft gluon emission,

  \[ 1/Q \]

  \[ 1/Q \]

  \[ 1/Q \]

  \[ 1/Q \]

  log$Q^2$ dependence

- Low $Q^2 \rightarrow 0$, hard gluon emission

  \[ 1/Q \]

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