Physics of Electrons in Two-Dimension

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In 2D, all electronic states are localized (scaling theory of localization)

1D — localized
2D — localized

Scaling theory of localization (disorder effect)

3D — minimum metallic conductivity
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In 2D, all electronic states are localized (scaling theory of localization)

Various energy scales:

- Thermal energy: $E_T = k_B T$ × low temperatures
- Fermi energy: $E_F = \frac{\pi h^2 n}{m^*}$
- Coulomb energy: $E_C = \frac{e^2 \sqrt{\pi n}}{\varepsilon}$ × high density
- e-e interaction
- Superconducting interaction
- Disorder

At low densities, Coulomb energy is dominant. ⇒ electron crystal (Wigner Crystal)
Search for Wigner crystal ......
electrons confined in semiconductor interface under high magnetic fields

(integer) quantum Hall effect

Electrons under magnetic fields \[ \vec{B} = B\hat{z}, \quad \vec{A} = -By\hat{x} \quad (\vec{B} = \nabla \times \vec{A}) \]

**Development of Landau level**

Hamiltonian, \[ \hat{H} = \frac{1}{2m} \left( \hat{p} - e \frac{A}{c} \right)^2 = -\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + \frac{1}{2m} \left( -i\hbar \frac{\partial}{\partial x} - eB \right)^2 \]

With a solution of the form \[ \Psi = \chi(y) e^{i(k_x x + k_z z)} \]

the Schrödinger equation, \[ \varepsilon \chi = \frac{\hbar^2 k_z^2}{2m} \chi + \frac{\hat{p}_y^2}{2m} \chi + \frac{1}{2} m \omega_c^2 (y - y_0)^2 \chi \quad \left( y_0 = \frac{\hbar c}{eB} k_x = \frac{\hbar k_x}{m \omega_c} \right) \]

simple harmonic motion in y-direction : \[ \varepsilon_n = \frac{\hbar^2 k_z^2}{2m} + \hbar \omega_c \left( n + \frac{1}{2} \right) \quad : \text{each energy level } \rightarrow \text{ Landau level} \]

**Level degeneracy** \[ \left\{ y_0 \text{ is the center of the harmonic oscillation} \right\} \quad 0 < y_0 = \frac{\hbar k_x}{m \omega_c} = \frac{\hbar}{m \omega_c} \frac{2\pi l_x}{L} < L \]

\[ \therefore 0 < l_x < \frac{m \omega_c L^2}{\hbar} \] \[ \therefore k_x = \frac{2\pi}{L} l_x \quad \text{from the boundary condition} \quad \Psi(x) = \Psi(x + L) \]

The number of oscillators at each level, \[ f = \frac{m \omega_c L^2}{\hbar} = \frac{eB L^2}{\Phi_0} = \frac{BL^2}{\Phi_0} = \frac{L^2}{2\pi B} \quad \left( \therefore \omega_c = \frac{eB}{mc}, \quad \frac{\hbar c}{e} = \Phi_0, BL^2 = \Phi \right) \]

**The level degeneracy increases with magnetic fields !**

At a higher field, each level can accommodate more electrons.

Less levels are filled with increasing fields !

Successive Landau level emptying (crossing) causes \[ \text{de Haas van Alphen oscillation} \]

**Shubnikov de Haas oscillation**
Question:
What happens at the exact field where n-th level is fully filled and (n+1)-th level is completely empty?

Generalizing the classical Hall effect, \( E_y = -\frac{B}{\rho \text{nec}} j_x = \rho_{yx} j_x \)

\[
\begin{pmatrix}
 j_x \\
 j_y 
\end{pmatrix}
=
\begin{pmatrix}
 \sigma_{xx} & \sigma_{xy} \\
 \sigma_{yx} & \sigma_{yy} 
\end{pmatrix}
\begin{pmatrix}
 E_x \\
 E_y 
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
 E_x \\
 E_y 
\end{pmatrix}
=
\begin{pmatrix}
 \rho_{xx} & \rho_{xy} \\
 \rho_{yx} & \rho_{yy} 
\end{pmatrix}
\begin{pmatrix}
 j_x \\
 j_y 
\end{pmatrix}
\]

\[\therefore \sigma_{xx} = \sigma_{yy} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}\]
\[\sigma_{xy} = -\sigma_{yx} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}\]

\[\text{no mobile electrons} \quad \therefore \sigma_{xx} = 0 \quad \rho_{xx} = 0\]

If \( \sigma_{xx} = \rho_{xx} = 0 \) then \( \rho_{xy} = -\frac{1}{\sigma_{xy}} = \frac{B}{\rho \text{nec}}\)

At exact level filling, zero longitudinal resistance and quantized Hall resistance.

**quantum Hall state**
Search for Wigner crystal …… continues…
with lower electron densities (cleaner sample) under high magnetic field

(integer) quantum Hall effect

Fractional quantum Hall effect


Stomer, H. L. Physica B 177, 401 (1992)

Single electron phenomenon

Collective phenomenon
Composite Fermion Picture

\[ \nu = \frac{3}{7} \]

\[ \frac{1}{2} \quad \frac{4}{9} \quad \frac{2}{5} \quad \frac{1}{3} \]

Fractional Quantum Hall states in electrons = Integer Quantum Hall states in composite Fermions

\[ \nu = 4 \quad 3 \quad 2 \quad 1 \]

Stomer, H. L. Physica B 177, 401 (1992)
Search for Wigner crystal …… continues… and continues… with even lower electron densities (even cleaner sample) under ZERO FIELD.

![Graphs showing resistance (ρ) vs. temperature (T) for different materials: Si-MOSFET and p-GaAs/AlGaAs.](image)

**Strongly Insulating**
\[
\left( \frac{d\rho}{dT} \ll 0 \right)
\]

**Decreasing Disorder**
\[
\left( \text{increasing density} \right)
\]

**Weakly insulating**
\[
\left( \frac{d\rho}{dT} \leq 0 \right)
\]

Abrahams, Anderson, Licciardello, and Ramakrishnan, PRL (1979)
(Scaling theory of localization)

Kravchenko et. al. (1995)
Sarachik and Kravchenko (1999)

Yoon, Shahar, Tsui, and Shayegan, PRL (1999)

**Unexpected metallic ground state!**
2D Fermi system

**Theory says ...** all 2D states are localized (insulators)

**But,** strong **Coulomb interaction** stabilizes a **metallic phase.**

2D Bose system

**Can superconducting interaction** stabilize a metallic phase?

**Theory says ...** **No. It is insulating** if the global superconductivity is suppressed.

Finkelshtein, LETP Lett. (1979)

Fisher, Grinstein, and Girvin, PRL (1990) : *“dirty boson” model*

**But,** experiments found a **metallic phase.**
How does the phase change occur in 2D superconductors?

Superconductor – Insulator Transition

- Resistance ($\rho$)
- Temperature ($T$)
- $T_c$
- Superconducting phase
- Insulating phase

Superconductor – Metal – Insulator Transition

- Resistance ($\rho$)
- Temperature ($T$)
- $T_c$
- Superconducting phase
- Metallic phase
- Insulating phase

Increasing $B$, or disorder