Why solid people need **fluid mechanics**?

Falkovich
WIS

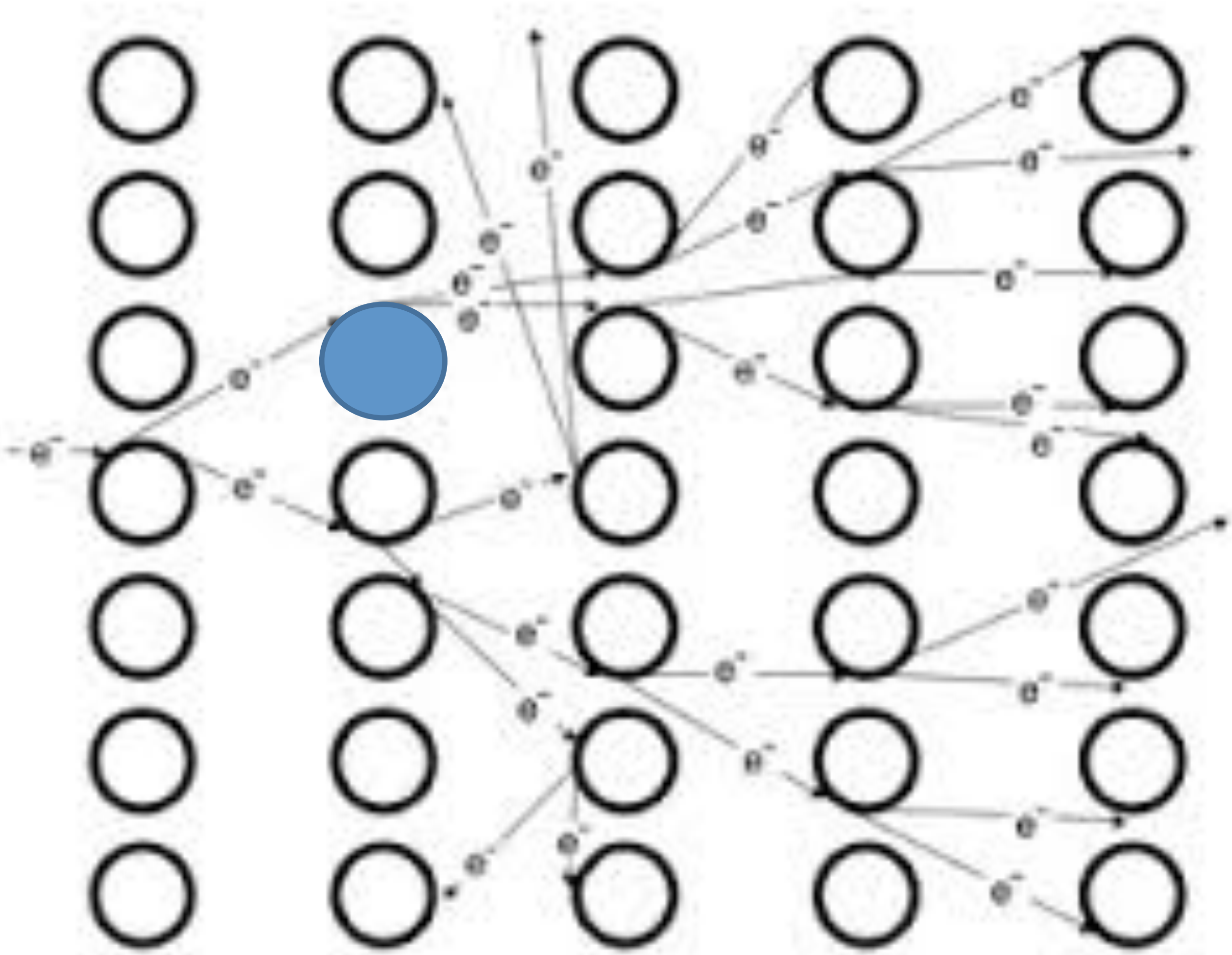
1.L Levitov & G Falkovich, **Electron viscosity, current vortices and negative nonlocal resistance in graphene.** *Nature Physics* **12** : 672-676 (2016)

2.G Falkovich & L Levitov, **Linking spatial distributions of potential and current in viscous electronics.** *Phys Rev. Lett. 2017, Arxiv:1607.00986 :*

3.H Guo, E Ilseven, G Falkovich, L Levitov, **Higher-than-ballistic conduction of viscous electron flows.** *PNAS 2017, Arxiv:1607.07269 ,1612.09239*

April 14, 2017 UVA

Electrons in a crystal lattice



Ohm's law: $j = nev$, $v = F\tau_p/m = e\nabla\varphi\tau_p/m$, $\boldsymbol{j = ne^2\tau_p/m\,\nabla\varphi}$

Is hydrodynamics ever relevant in metals?

In one-component fluid or gas a hydrodynamic approach works because one has local conservation of energy and momentum. Macroscopic hydrodynamic equations describe propagation of conserved quantities in space.

Electron fluid in a solid can exchange energy and momentum with the lattice. Hydrodynamics not relevant?

If the disorder scattering time τ_p exceeds the electron-electron scattering time $\tau_{ee} = l / v_F$, then electrons behave as viscous liquid.

High-mobility electron systems (GaAs 2DES, graphene). Non-Fermi liquids, high-Tc superconductors, strange metals.

$$\gamma_{ee}^{-1} \approx 80\,\mathrm{fs} \qquad \gamma_{p}^{-1} \sim 0.5\,\mathrm{ps}$$

Fritz, L., Schmalian, J., Müller, M. & Sachdev, S. Quantum critical transport in clean graphene. Phys. Rev. B **78**, 085416 (2008).

Kashuba, A. B. Conductivity of defectless graphene. Phys. Rev. B 78, 085415 (2008).

Kinematic viscosity - diffusivity of momentum

$$\nu \approx v_F^2 \gamma_{ee}^{-1} \approx v_F l_{ee}$$

$$\nu \approx v l$$

typical velocity   mean free path

water	$\nu = 0.01\ cm^2\ sec^{-1}$
air	$\nu = 0.15\ cm^2\ sec^{-1}$
electrons in graphene	$\nu = 1000\ cm^2\ sec^{-1}$

Dimensionless coupling constant

In graphene (or any other Dirac material), the strength of electron-electron interactions is controlled by the dimensionless parameter, called "fine structure constant" (because of its analogy with the QED fine structure constant):

$$\alpha_{\rm ee} = \frac{e^2/(\epsilon L)}{\hbar v_{\rm F}/L} = \frac{e^2}{\epsilon \hbar v_{\rm F}}$$

This dimensionless number is:

1) **not** small, i.e. it is of order unity
2) **not** gate tunable (Fermi wave number drops out)
3) **sensitive** to dielectric environment (the "epsilon factor")

V. N. Kotov *et al.*, Rev. Mod. Phys. **84**, 1067 (2012)

The main problem:

WHAT IS THE MOST FUNDAMENTAL MANIFESTATION OF STRONGLY INTERACTING ELECTRON FLOW?

Hydrodynamic description of transport

$$\gamma_{\mathrm{p}} \ll \gamma_{\mathrm{ee}}$$

Navier-Stokes equation

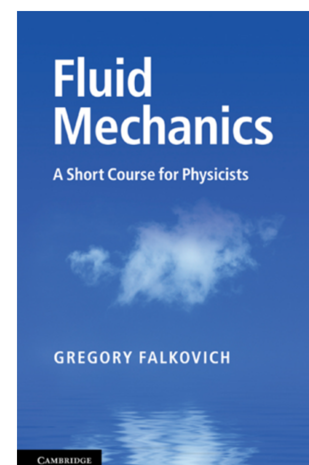
$$\partial_t v + (v\nabla)v - \nu\nabla^2 v = -\nabla P/mn$$

$$\nu \approx (1/2)v_{\mathrm{F}}^2\gamma_{\mathrm{ee}}^{-1}$$

$$P = e\int_{n_0}^{n} \Phi(n')\mathrm{d}n'$$

$E_{\mathrm{F}} \gg k_{\mathrm{B}}T$

$$P \approx e(n - n_0)\Phi$$



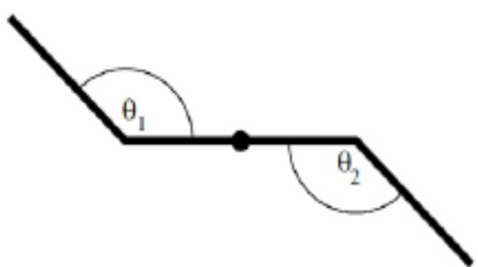
Life at low Reynolds numbers

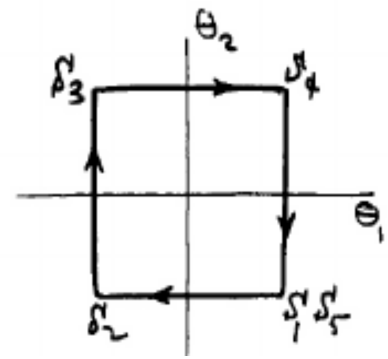
$Re = vL/\nu << 1$ $\eta \Delta v = \nabla P$

●**Scallop theorem** (Purcell 1977) to achieve pumping or propulsion at low Reynolds number one must deform in a way that is not invariant under time-reversal.

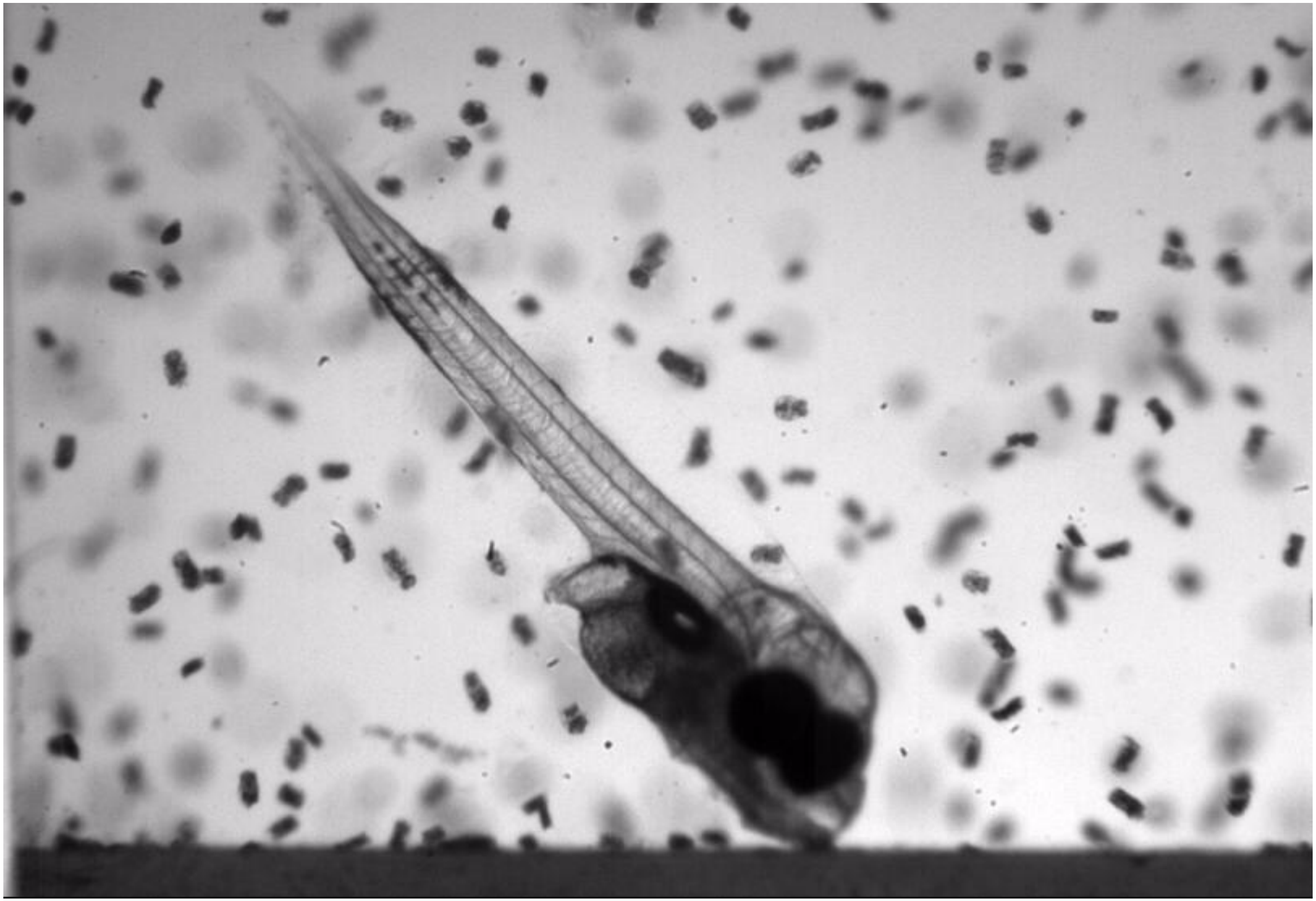
●**Berry phase & non-abelian gauge theory:** Wilczek, Shapere (1989), Geometry of self-propulsion at low *Re*. Avron, Kenneth, Gat (2004).

Swimming (pumping) consists in periodically changing shape to move relative to the fluid





Life and death at low Reynolds number



Microfluidic flows and Ohmic-viscous currents

Consider plane viscous flow between two plates separated by *h*.

$$\boldsymbol{v}(x,y,z)=6z(h-z)\boldsymbol{v}(x,y)$$

$$\frac{1}{h}\int_0^h dz(\eta\Delta v-\nabla P)=\eta\Delta_\perp v-\frac{12\eta}{h^2}v-\nabla P=0$$

$$\eta\Delta_\perp v-\rho(ne)^2 v-ne\nabla\varphi=0$$

Ohmic resistance

Purely Ohmic case

Consider incompressible flow $\nabla \cdot \mathbf{j} = ne\nabla \cdot \mathbf{v} = 0$

$$\mathbf{v} = \mathbf{z} \times \nabla\psi = (-\partial_y\psi, \partial_x\psi)$$

$$\mathbf{j} = en\mathbf{v} = -\sigma\nabla\phi$$

$$en\mathbf{z} \times \nabla\psi = -\sigma\nabla\phi$$

Cauchy-Riemann conditions

$$\Delta\psi = \Delta\varphi = 0$$

Purely viscous case
Current and potential in viscous electronics

$$\eta \nabla^2 v_i = ne \nabla_i \phi, \quad \nabla_i v_i = 0$$

vorticity $\omega = \nabla \times \mathbf{v} = \mathbf{z} \nabla^2 \psi$ is non-zero

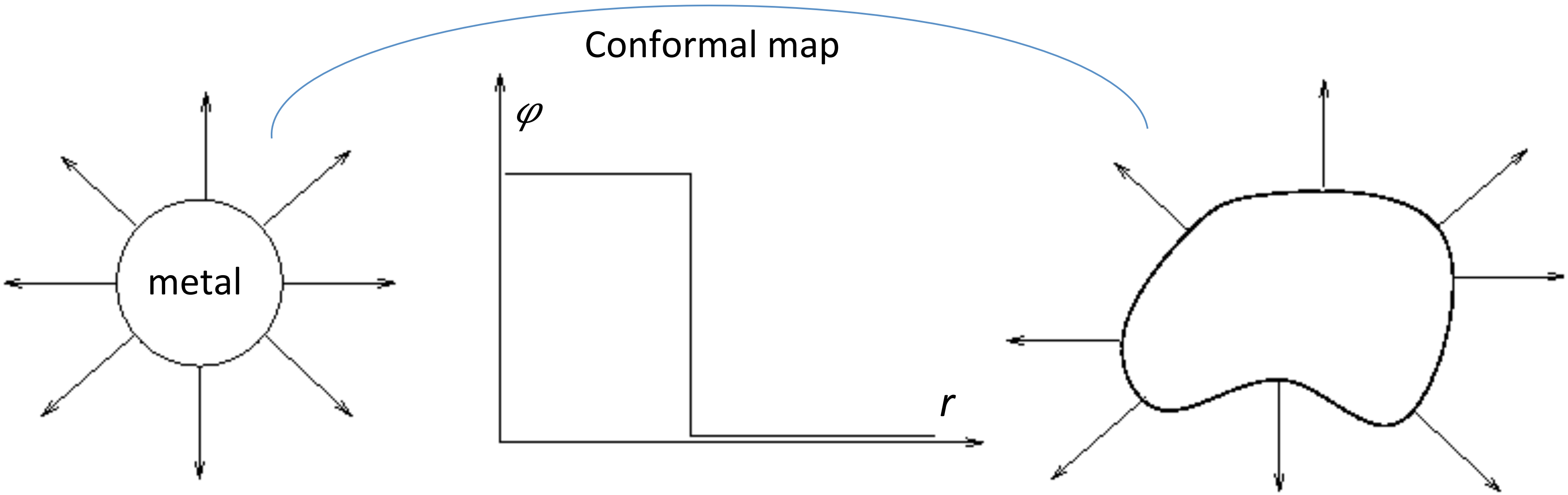
$$\partial_x \omega = (en/\eta) \partial_y \phi, \quad \partial_y \omega = -(en/\eta) \partial_x \phi$$

Δω=Δ↑2 ψ=Δφ=0

Field expulsion from viscous current flow

$$\omega + i\varphi - analytic\ function$$

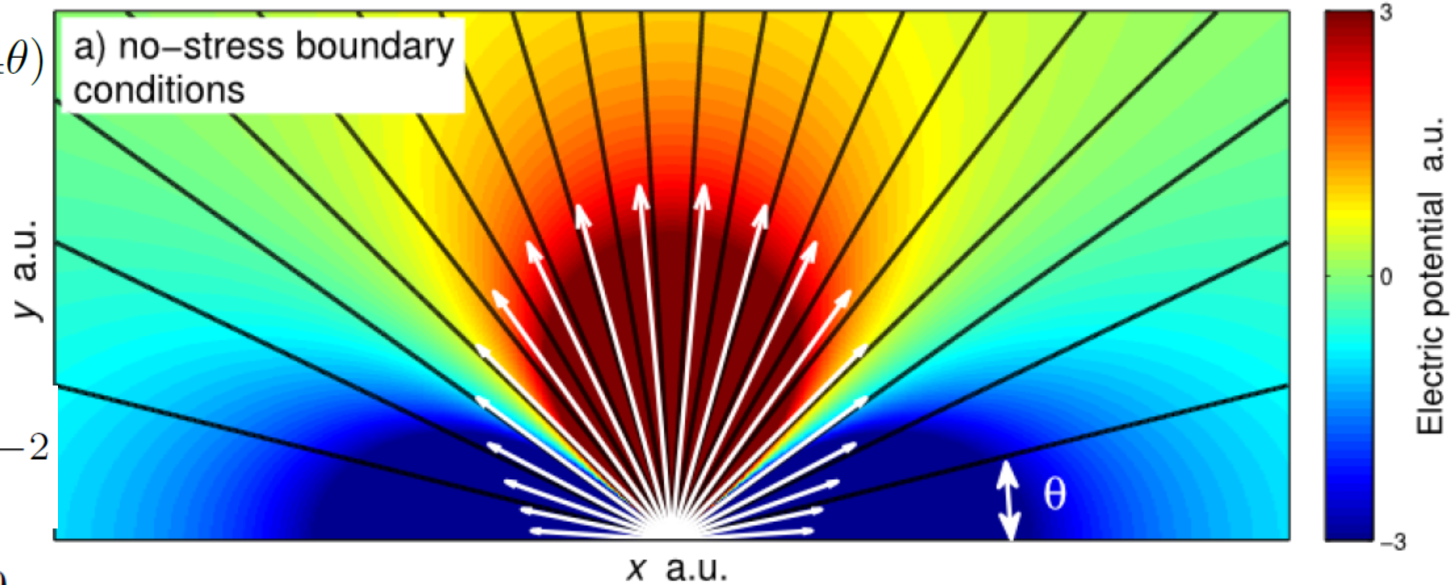
$$v \propto 1/r \rightarrow \Delta v = 0 \rightarrow \omega = 0,\ \varphi = const$$



The simplest non-trivial viscous current flow

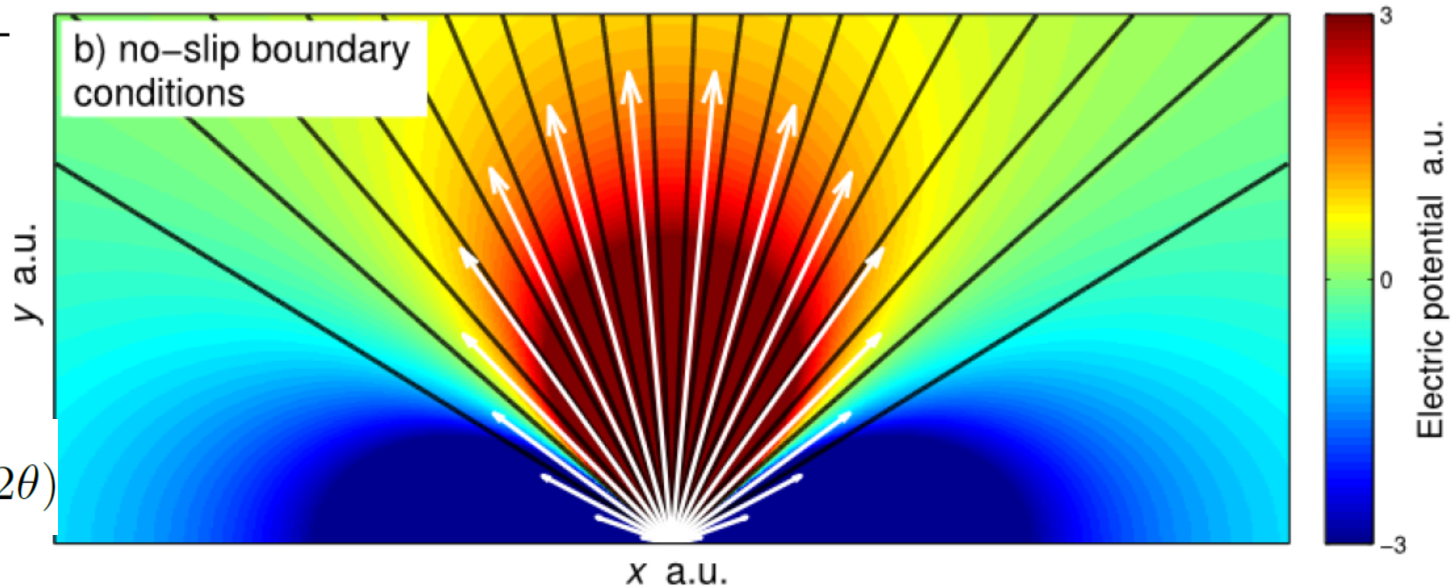
(non-metallic boundary)

$$\psi_1(\theta) = \frac{\tilde{I}}{4\pi}(\sin 2\theta - 4\theta)$$



$$\phi(x, y) = \frac{\tilde{I}\eta}{2ne} \operatorname{Re} z^{-2}$$

$$= -\frac{\tilde{I}\eta}{2ne} \frac{\cos 2\theta}{r^2}$$



$$\psi_2(\theta) = \frac{\tilde{I}}{2\pi}(\sin 2\theta - 2\theta)$$

Negative voltage

IS THE MOST FUNDAMENTAL MANIFESTATION OF STRONGLY INTERACTING ELECTRON FLOW?

DC viscous flow

$$\eta\nabla^2 v_i = ne\nabla_i\phi, \quad \nabla_i v_i = 0,$$

$$\text{vorticity } \omega = \nabla\times\mathbf{v} = \mathbf{z}\nabla^2\psi \text{ is non-zero}$$

Purely viscous case

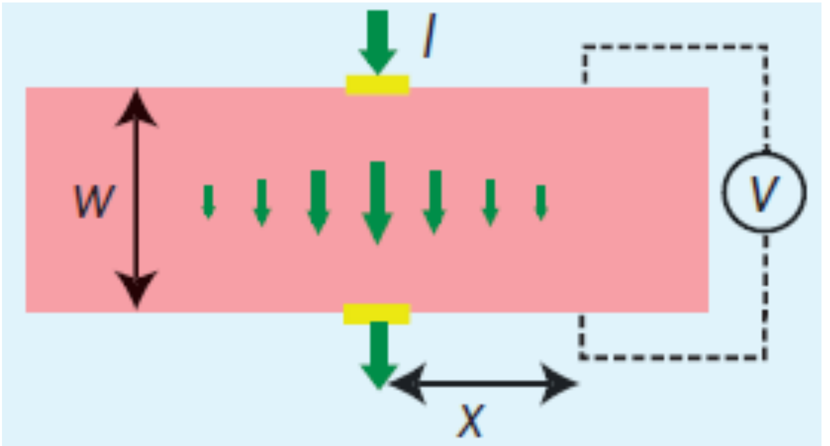
$$(\nabla^2)^2\psi = 0$$

Mixed Ohmic-viscous case

$$[\eta(\nabla^2)^2 - \rho(en)^2\nabla^2]\psi(x,y) = 0$$

$\rho = \gamma_p m/ne^2$ is resistivity

$$v_y(x,y)_{y=0,w} = \partial_x\psi(x,y)_{y=0,w} = \frac{I(x)}{en}$$



general boundary condition

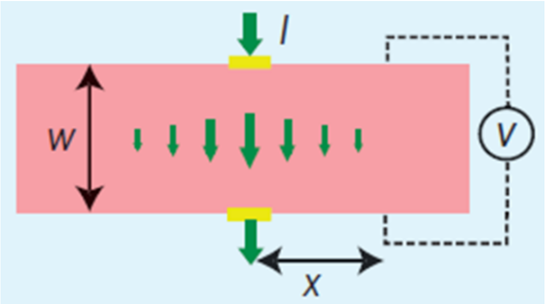
$$v_\perp = 0, \quad v_\parallel = -\alpha\partial_\parallel P$$

DC viscous flow in a strip

$$\eta\nabla^2 v_i = ne\nabla_i\phi, \quad \nabla_i v_i = 0$$

$$(\nabla^2)^2\psi = 0 \qquad \nabla_l\psi = \frac{I(r)}{en} \qquad \nabla_n\psi = 0$$

$$\psi(x,y) = \bar{z}f(z) + g(z)$$



$$\psi(x,y) = \int\limits_{-\infty}^{\infty} \frac{dk}{2\pi}\frac{I(k)e^{ikx}}{neik}\left\{\frac{\cosh k(y-w/2)}{\cosh kw/2}\right.$$

$$\left. + \frac{k\tanh kw/2}{kw+\sinh kw}[y\sinh k(w-y) + (w-y)\sinh ky]\right\}$$

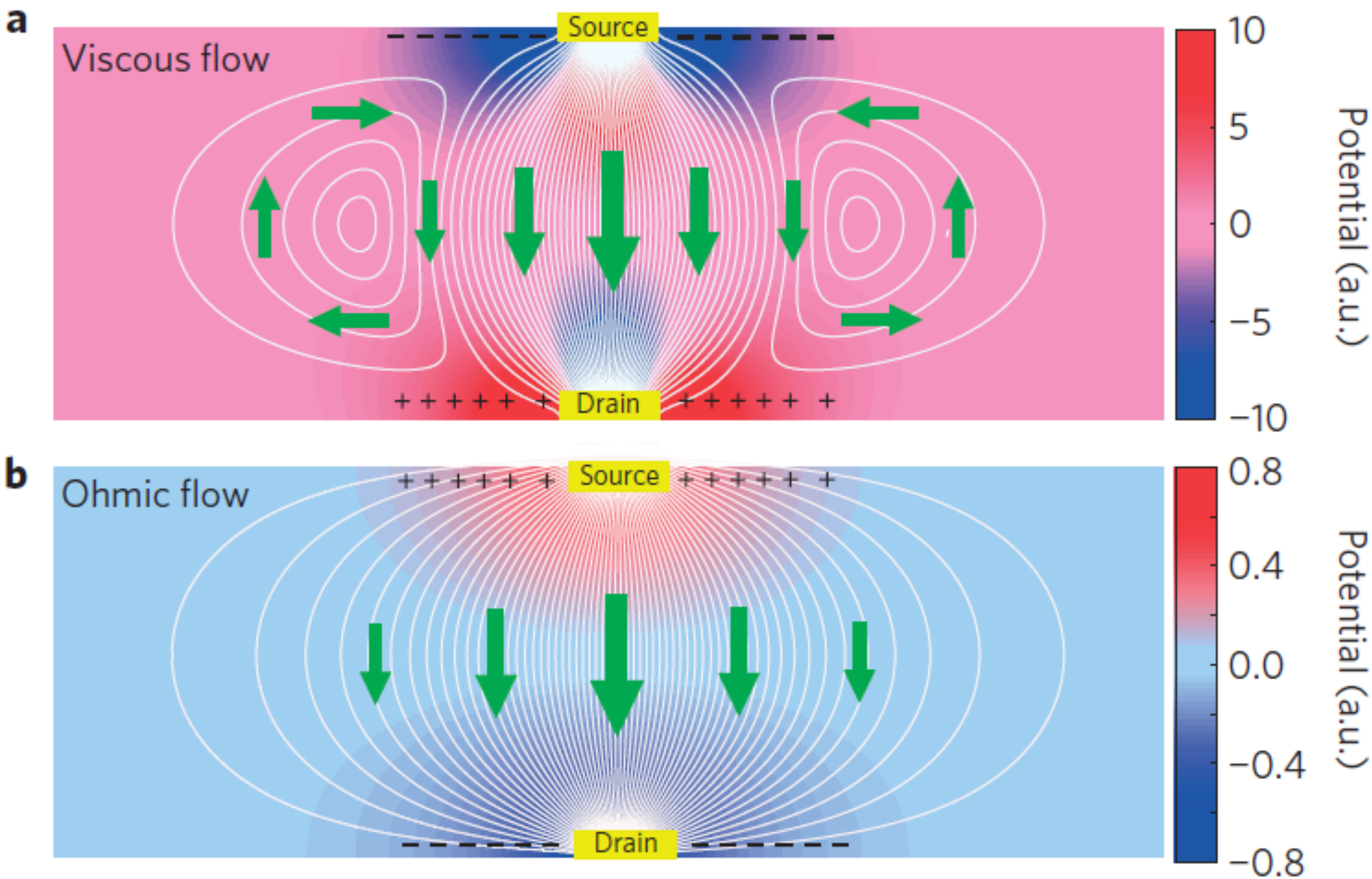
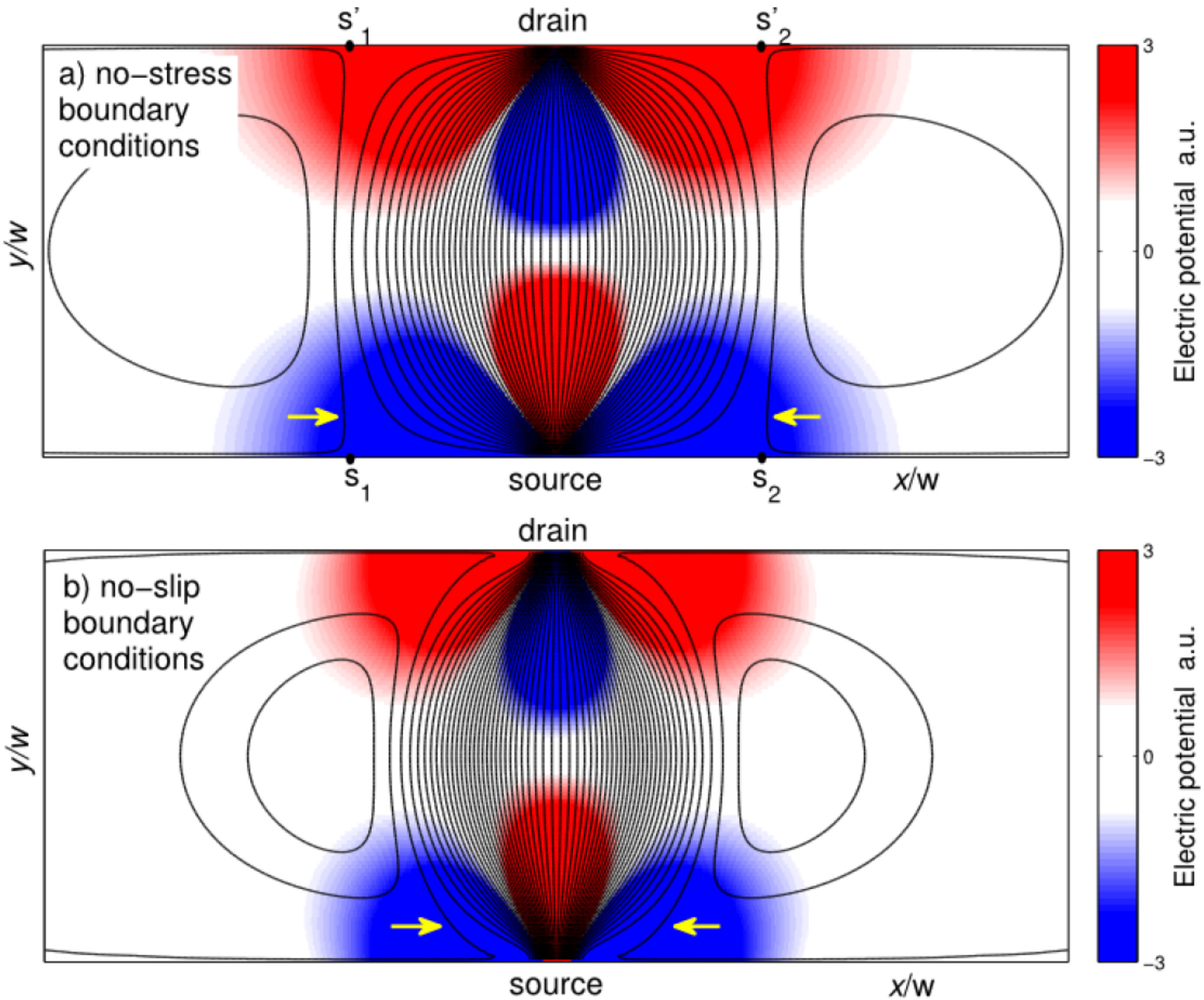


Figure 1 | Current streamlines and potential map for viscous and ohmic flows. White lines show current streamlines, colours show electrical potential

Comparing no-slip and no-stress cases



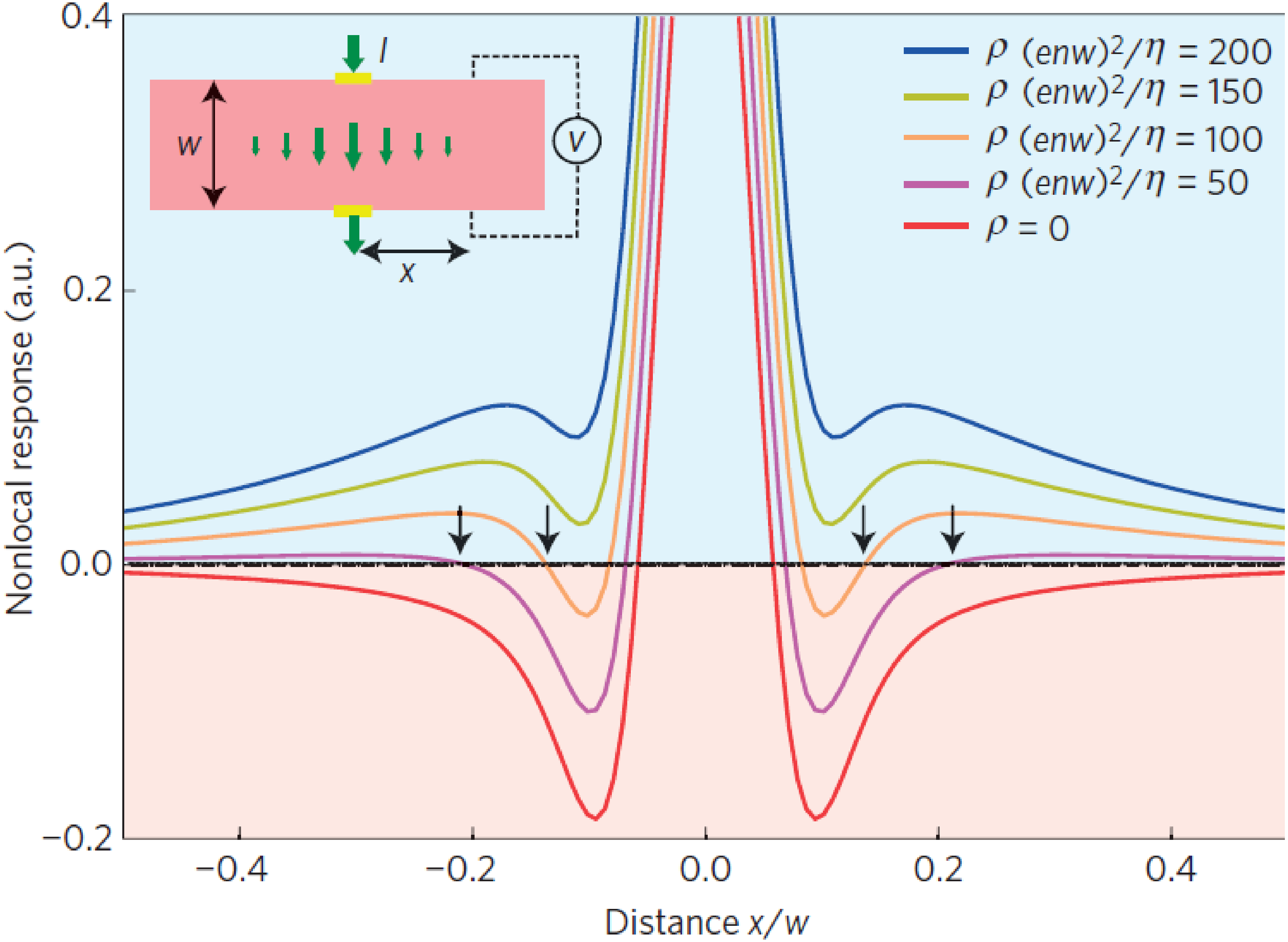


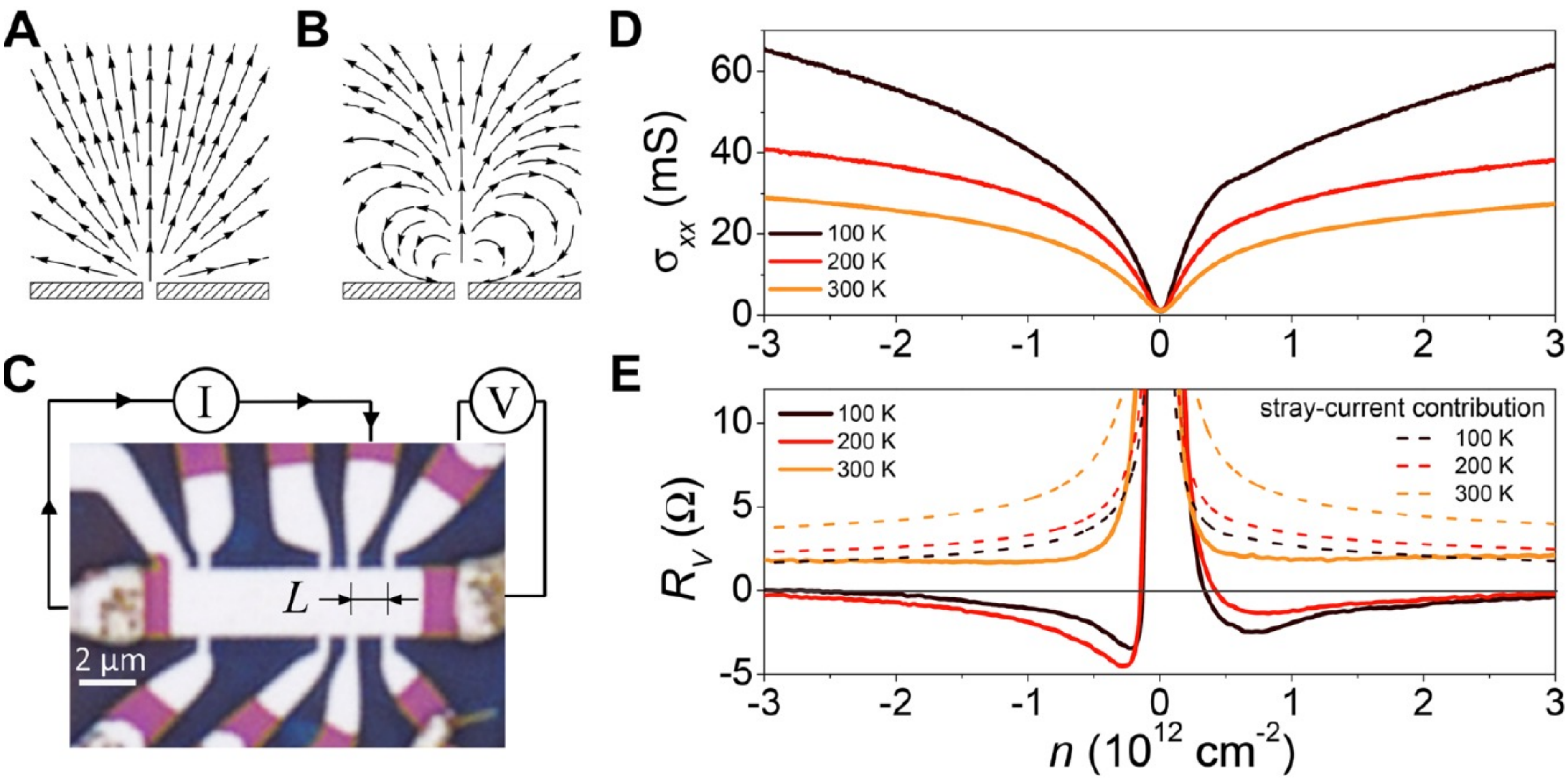
Figure 2 | Nonlocal response for different resistivity-to-viscosity ratios $\boldsymbol{\rho/\eta}$

$$\epsilon = \rho(enw)^2/\eta \approx 2\gamma_{ee}\gamma_p (w/v_F)^2$$

Negative local resistance caused by viscous electron backflow in graphene

Science
11 February 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]



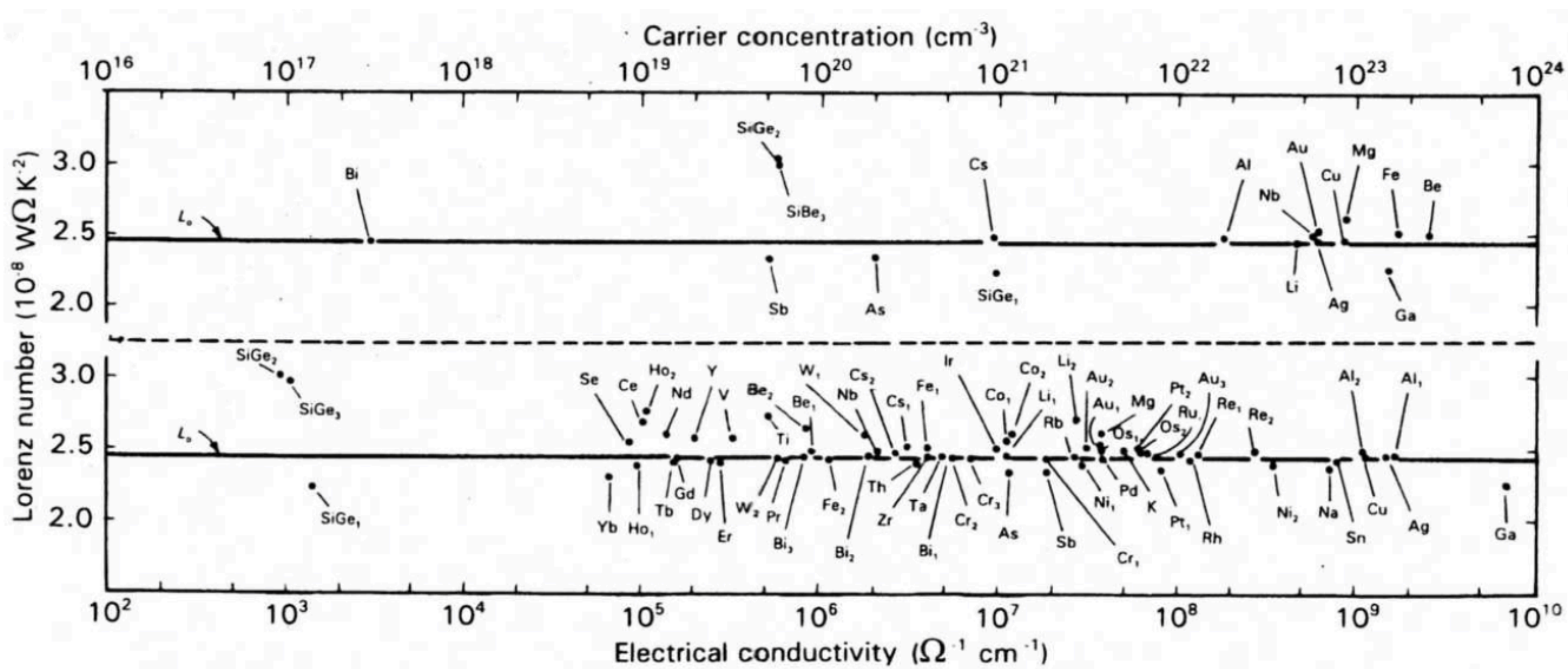
I = 0.3 μA; L = 1 μm; W = 2.5 μm

Measured viscosity ≈ 0.1 m^2 s^{-1}

Wiedemann-Franz law

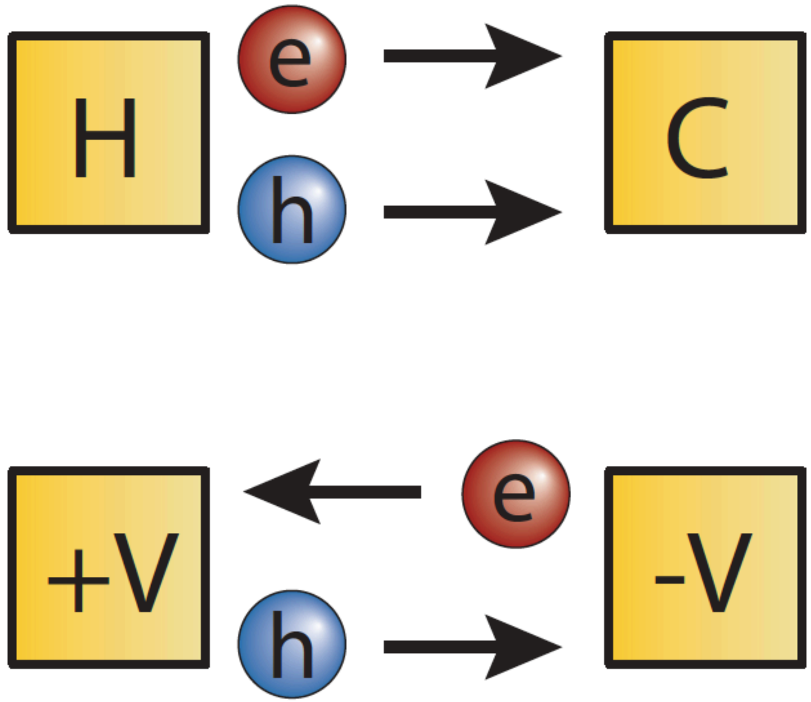
$$\frac{\kappa_{WF}}{\sigma_{elec} T} = L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

- Independent of density, mass, mean-free-path, scattering time
- True in Drude model and Fermi liquid



Kumar, Prasad, Pohl, J. of Materials Sci. 28, 4261 (1993)

Experimental signature of Dirac fluid



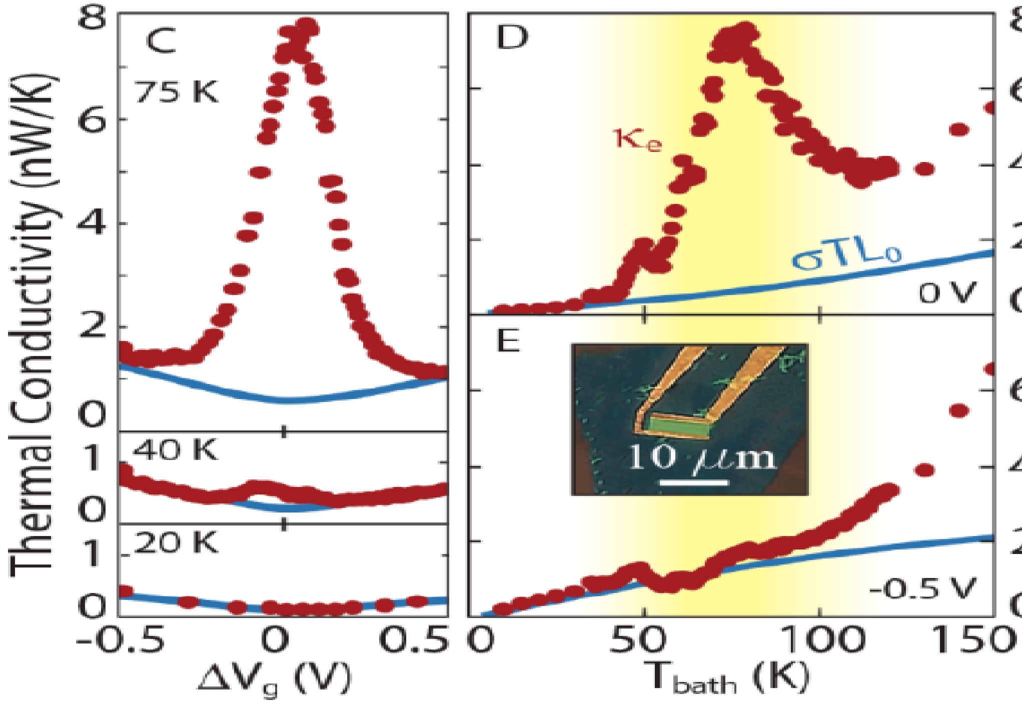
$$\frac{\kappa_{DL}}{\sigma_{elec} T} > L_0$$

Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Science
10.1126/science.aad0343 (2016)

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

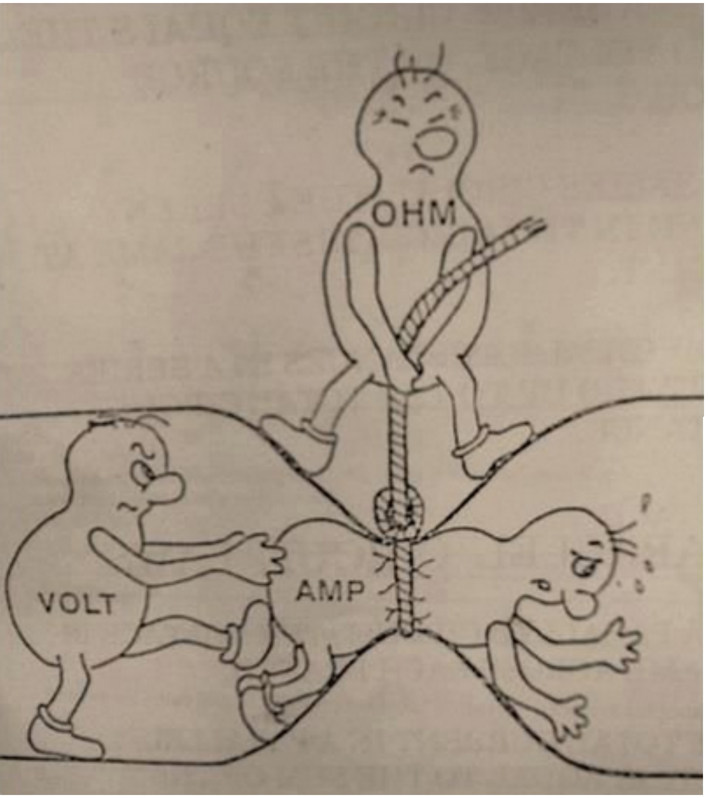
Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point, which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, thanks to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report an order of magnitude increase in the thermal conductivity and the breakdown of the Wiedemann-Franz law in the thermally populated charge neutral plasma in graphene. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.



How interaction between carriers of the same sign changes resistance? Does the resistance in the viscous regime always exceed that in the Ohmic regime?

One must distinguish between Ohmic resistance due to scattering on
a) phonons, b) impurities or boundaries.

Exceeding ballistic conductance in viscous flows

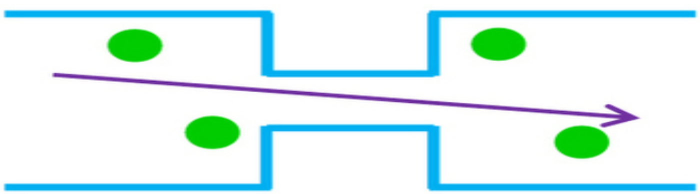


ballistic free-fermion

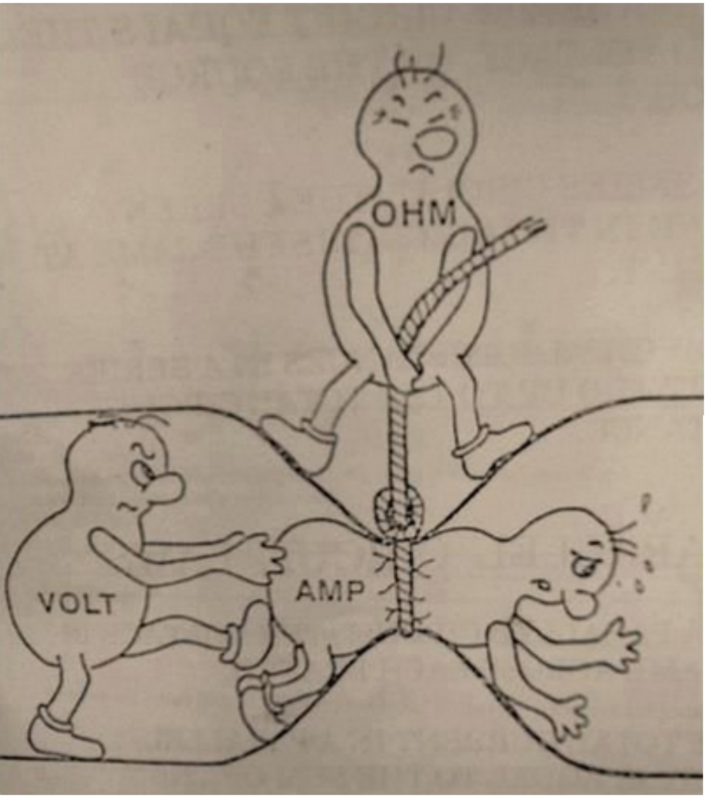
$$R_{\rm ball} \;=\; \frac{h}{2e^2} N^{-1}$$

$$N \;\approx 2w/\lambda_F$$

a



Exceeding ballistic conductance in viscous flows



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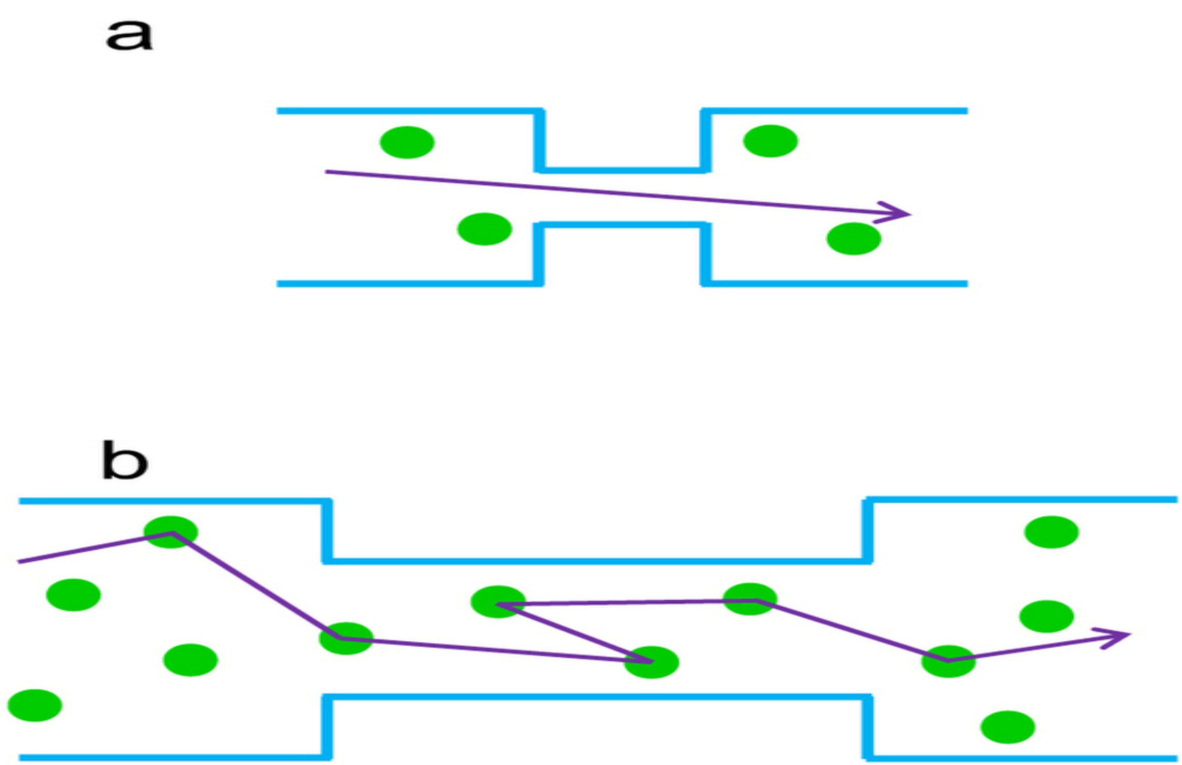


viscous resistance

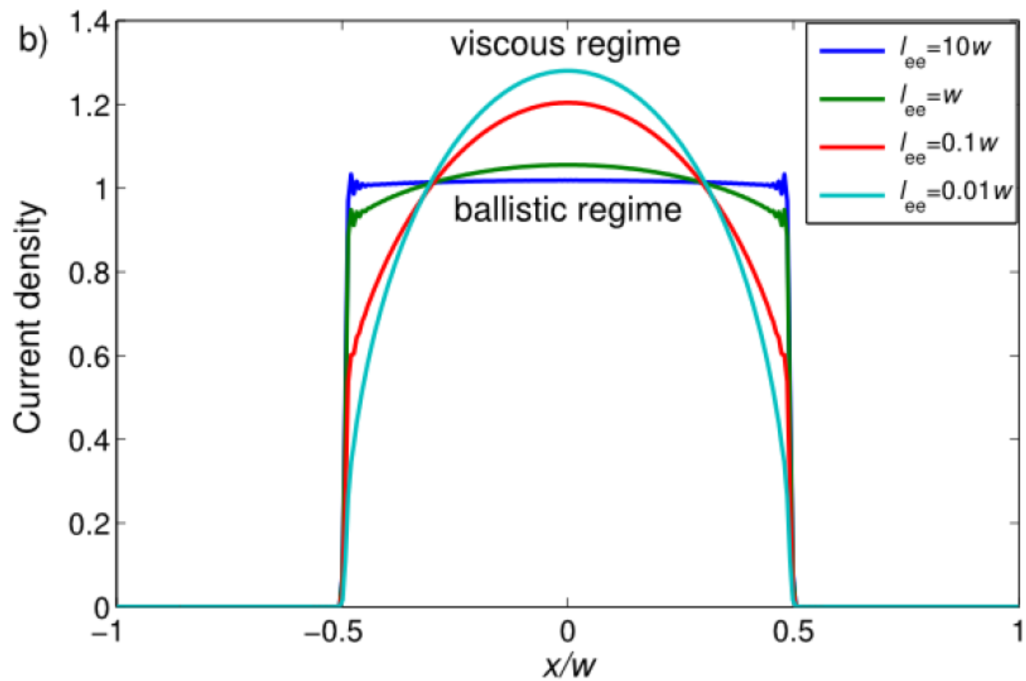
$$R_{\text{vis}}(w) = \frac{32\eta}{\pi(ne)^2 w^2}$$

Exceeding ballistic conductance in viscous flows

ballistic free-fermion

$$R_{\rm ball} = \frac{h}{2e^2} N^{-1}$$

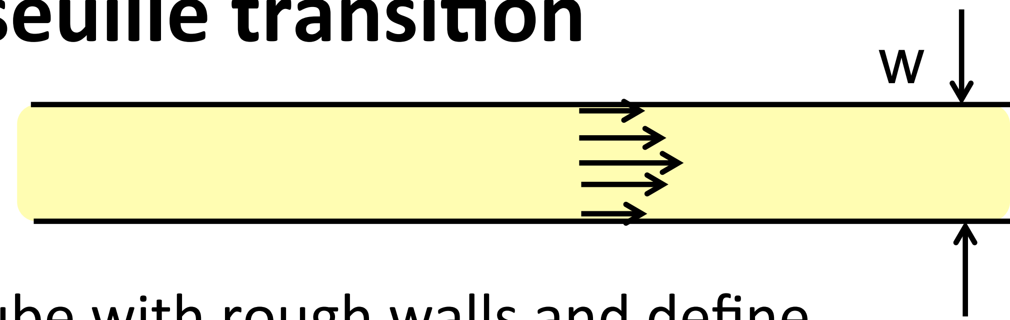
$$N \approx 2w/\lambda_F$$



$$R_{\rm vis}(w) = \frac{32\eta}{\pi(ne)^2 w^2}$$

it is easier for **interacting** electrons to go through the eye of a needle…

Knudsen-Poiseuille transition

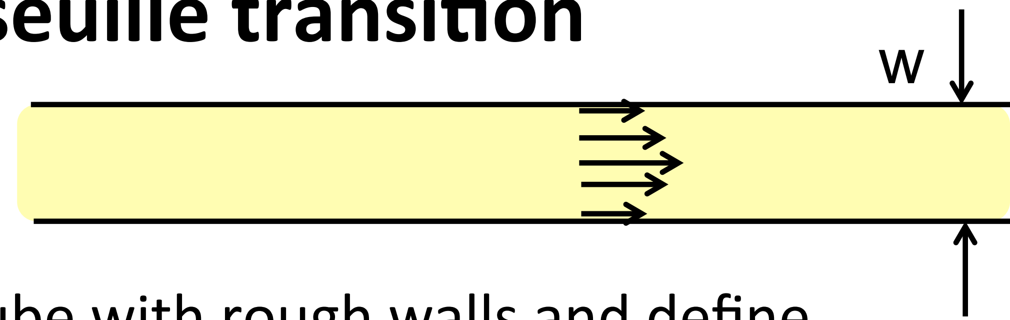


Consider gas flowing through a tube with rough walls and define resistance as force divided by momentum: $R \equiv 1/\tau \equiv \nabla P/mnU$, analog $R \equiv V/I$

Knudsen regime $\mathrm{w} < l$, $\tau \sim w/v_T$.

Poiseuille regime $\mathrm{w} > l$, $\tau \sim w^2/\nu \sim w^2/v_T l = w/v_T \, w/l$

Knudsen-Poiseuille transition



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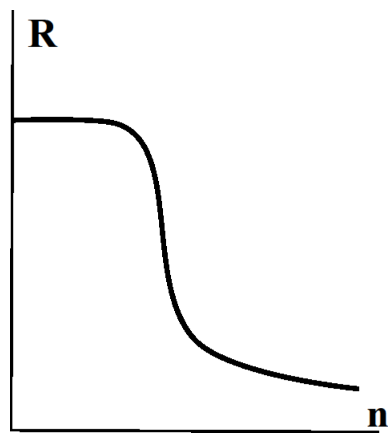
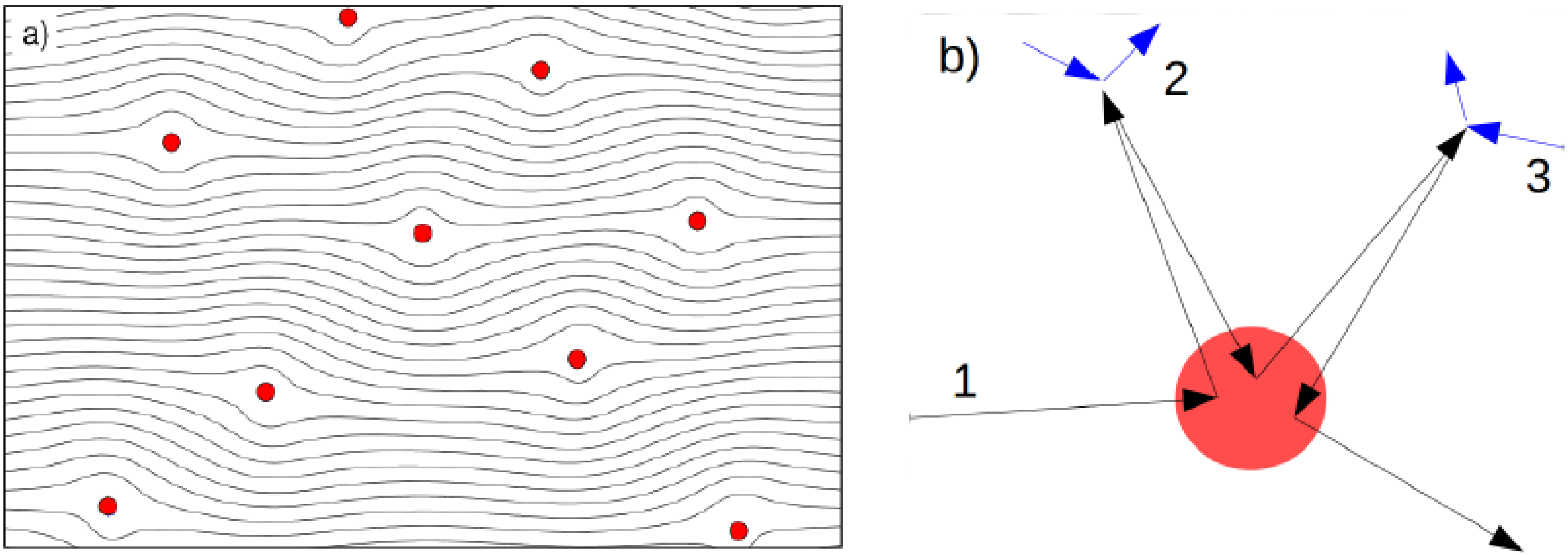


Stokes Paradox, Back Reflections and Interaction-Enhanced Conduction

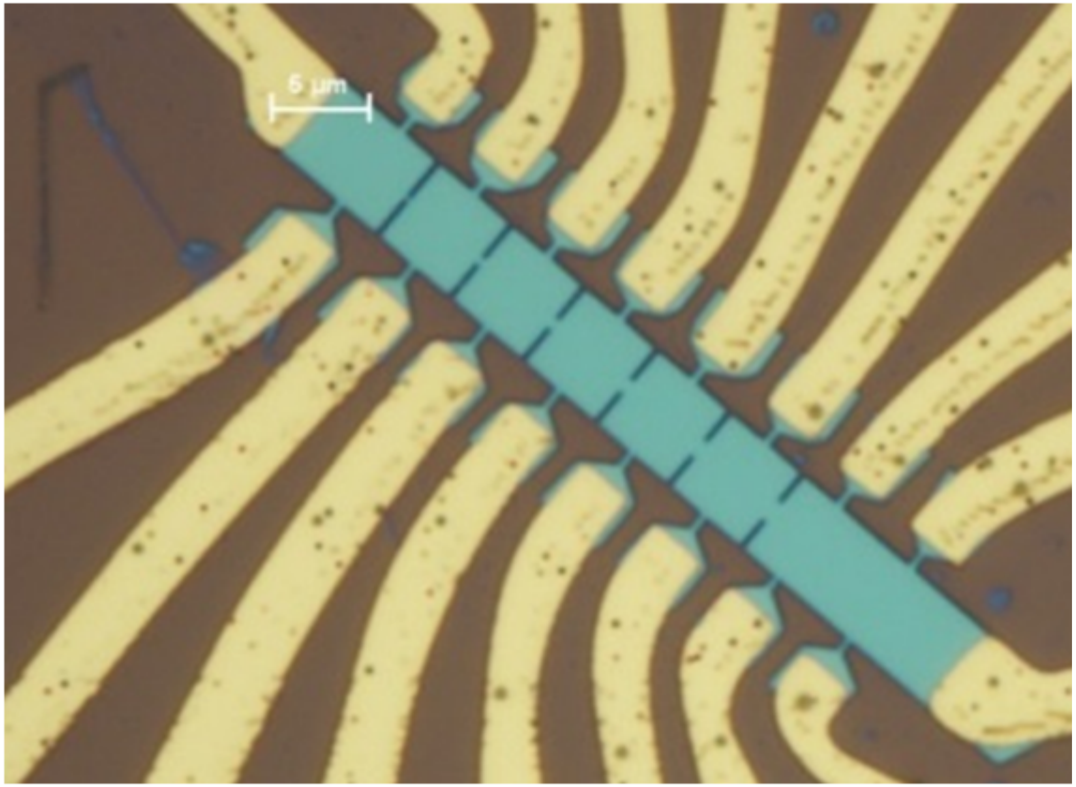


$$\sigma = \frac{Ne^2 v_F^2}{2n_s}\left(\frac{1}{U_0} + \frac{1}{4\pi\nu}\ln\frac{L}{a_*}\right)$$

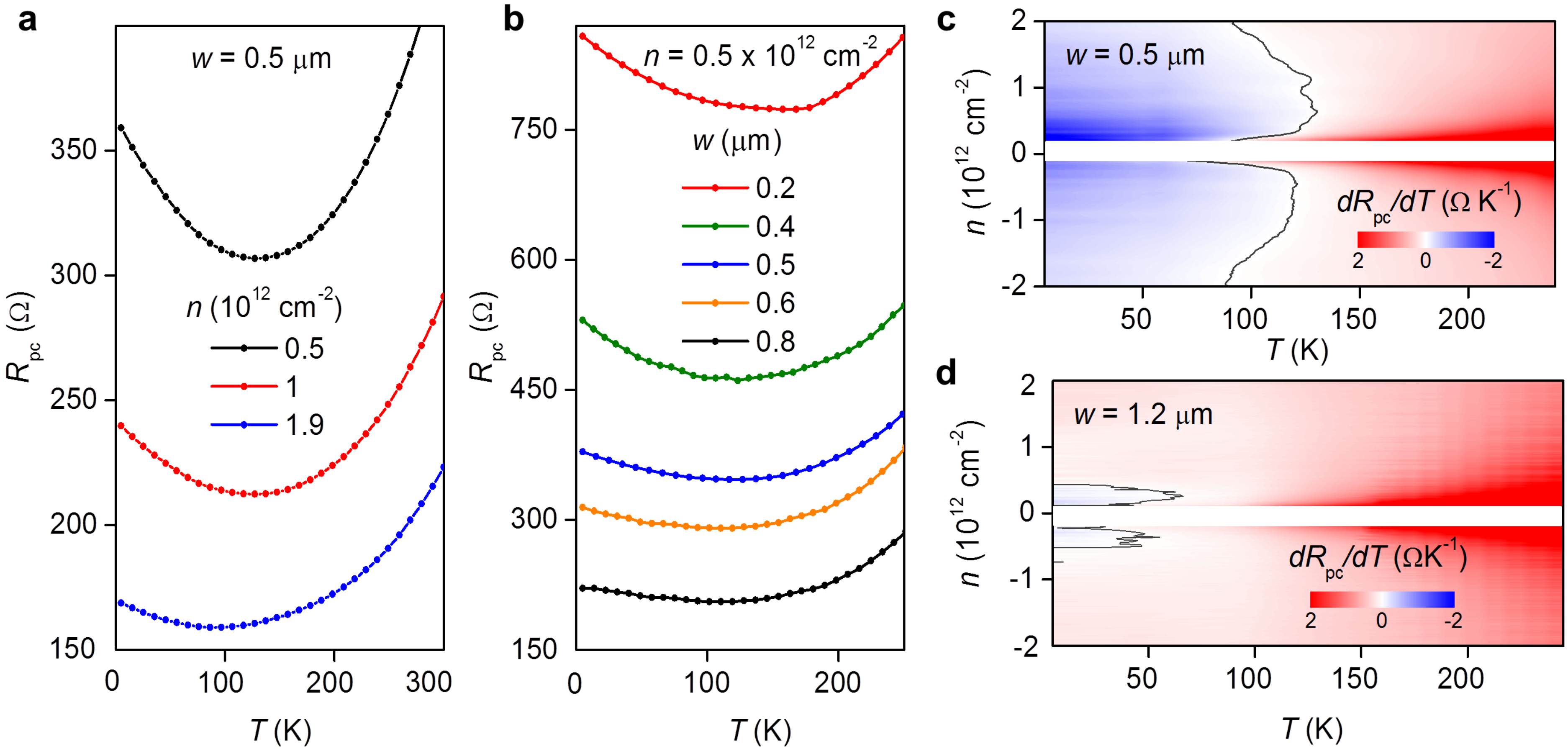
$$a_* = (a\ell_{\mathrm{ee}})^{1/2}$$

H Guo, E Ilseven, G Falkovich and L Levitov, *PNAS* 2017, [Arxiv:1607.07269](https://arxiv.org/abs/1607.07269)

Experiment



R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y Cao, A Principi, H Guo, G Auton, M Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim, submitted to Nature Physics



R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y. Cao, A. Principi, H. Guo, G.H. Auton, M. Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim, submitted to Nature Physics

$$R_{\rm pc} = \frac{1}{G_{\rm b} + G_\nu} + R_{\rm C}$$

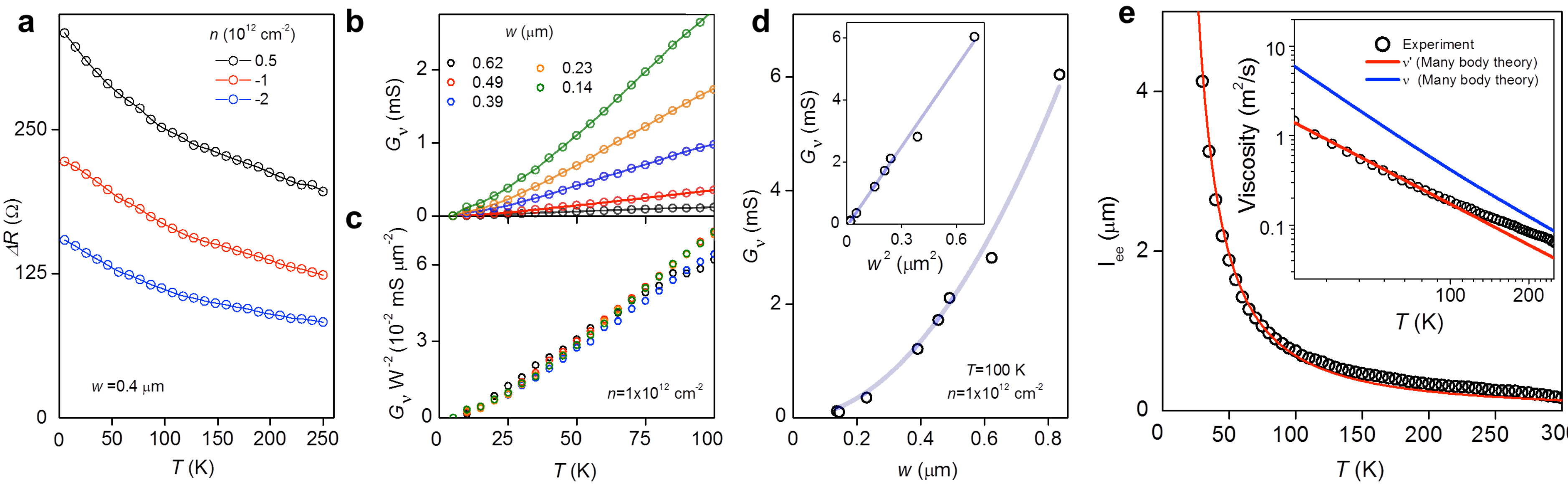
Sharvin (ballistic)

e-e interaction contribution

“contact resistance” (diffusive contribution)

Guo,Ilseven,Falkovich,Levitov,
PNAS March 2017
arXiv:1607.07269,1612.09239

$$G_\nu = \frac{\sqrt{\pi|n|}e^2 w^2 v_{\rm F}}{32\hbar\nu}$$



R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y. Cao, A. Principi, H. Guo, G.H. Auton, M. Ben Shalom, L.A. Ponon
G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim, submitted to Nature Physics

So where was the cheating?

So where was the cheating?

If there is no cheating, then it is not theoretical physics but mathematics.

Moral:

viscosity makes current-voltage relation nonlocal opening new possibilities; Strongly interacting electrons can flow like a laminar viscous flow and demonstrate negative resistance, super-ballistic conductance and other wonders.

Future "viscous electronics" needs *fluid mechanics*